Toward Geometric-Topological Memory for AI: A Research Program Companion to QnapsiZ

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Abstract

This paper develops the theoretical research program underlying QnapsiZ, a geometric-topological external memory architecture for AI systems. The companion systems paper (a comprehensive 24-page manuscript with 10 experiments) presents the implemented stack: volumetric concept storage, topology-aware ingestion and retrieval, portalized connectomes, replay-driven persistence, and controlled proof-of-concept evaluation. The present manuscript has a different goal. We separate implemented mechanisms from the broader conjectures they motivate and ask which of those conjectures can be made mathematically explicit, empirically falsifiable, and architecturally useful. We argue that persistent geometric state is not merely an engineering convenience but a candidate explanatory layer for memory organization, contradiction detection, and hierarchical knowledge routing. We formulate open hypotheses about homological grounding, structure-before-specificity, manifold-locality, and multi-connectome memory, while also identifying the limits of the current evidence. The result is not a claim of completed theory, but a scaffold for a program in which geometry and topology become operational substrates for AI memory rather than metaphors retrofitted onto token retrieval.

Keywords— AI Memory, Information Topology, Geometric External Memory, Persistent Homology, Sparse Volumetric Computing, Memory Hierarchies, Research Program.

1 Introduction

Current memory systems for large language models are powerful but flat. Even when they use dense vector stores, key-value memories, graph traversal, or retrieval-augmented generation pipelines, they usually treat memory as lookup over discrete artifacts rather than as a persistent state with its own inspectable geometry. This is an effective engineering choice for many applications. It may also be a conceptual bottleneck. If memory remains only a sparse search layer around a token predictor, then contradiction, drift, interference, and confidence remain phenomena inferred indirectly from model behavior rather than properties measured on the memory substrate itself.

QnapsiZ explores a different possibility. In the companion systems paper, concepts are externalized as localized density fields in a voxelized connectome; ingestion modulates those fields through topological and geometric operations; retrieval samples them through ray marching, local homology checks, snapping, and portalized traversal across manifolds. That paper asks whether such a system can be built and whether it behaves coherently under controlled conditions. This paper asks a different question: what kind of theory would make a geometric memory architecture more than a bundle of useful heuristics?

We do not claim to answer that question completely. The current implementation does not validate a general theory of grounded cognition. It does not prove that topological signals are sufficient for truth maintenance. It does not establish that cognitive analogies imported into the QnapsiZ vocabulary are mechanistic facts. What it does provide is a concrete implementation from which more precise theoretical questions can be extracted. That extraction is the purpose of this paper.

The central claim of the present manuscript is modest and structural. We argue that three lines of inquiry deserve to be treated together rather than separately. The first concerns *persistent geometric state*: whether memory representations gain new diagnostic and control affordances when they become spatial objects rather than retrievable text fragments. The second concerns *topological diagnostics*: whether local and global homological properties can serve as operational signals of contradiction, interference, or semantic multiplicity. The third concerns *hierarchical manifold memory*: whether nested connectomes and portalized routing provide a mathematically cleaner account of domain separation, collision handling, and expert memory than flat retrieval spaces.

The rest of the paper is organized accordingly. Section 2 draws a strict boundary between the systems paper and the theory paper. Section 3 introduces the conceptual substrate of geometric external memory. Section 4 treats structure-before-specificity and CCUP-style reasoning as a formal research direction rather than an established law. Section 5 formulates the notion of homological grounding and its open problems. Section 6 revisits the Gestalt-Locality hypothesis in falsifiable terms. Section 7 addresses hierarchical manifold memory. Section 8 situates the paper's large analogies as heuristic scaffolds, not proofs. Section 13 offers concrete predictions that can guide future experiments.

2 Boundary With The Systems Paper

This companion paper exists because the systems paper and the theory program are not the same evidentiary object. The systems paper is judged by whether the architecture exists, whether the mechanisms run, whether the benchmarks are honestly scoped, and whether the implementation can be evaluated against alternatives. The theory paper is judged by a different standard: whether it states clear hypotheses, defines terms carefully, distinguishes analogy from derivation, and identifies what would count as confirmation or refutation.

The distinction matters because the language surrounding QnapsiZ naturally invites conceptual overreach. Terms such as "living connectome," "right brain," "grounding," and "topological truth" can function either as productive research prompts or as rhetorical shortcuts. In the systems paper, these phrases must be subordinated to the implementation and the measurements. In the theory paper, they can be retained only if they are disciplined into definitions, conjectures, or testable hypotheses.

We therefore impose the following division of labor. The systems paper owns implementation claims. It also owns benchmark claims. This paper owns broader interpretive and mathematical questions, but only in conjectural form unless external proof or decisive experiment is available. When a mechanism has been deployed in code, we say so. When a mathematical bridge remains incomplete, we say so. When an analogy is used to orient the reader rather than to derive an operator, we mark it explicitly as analogy.

3 Conceptual Foundations

3.1 Geometric External Memory

External memory for AI is usually discussed in computational rather than ontological terms: where information is stored, how it is indexed, what cost model retrieval obeys, and how the host model consults it. Geometric external memory introduces a different emphasis. It treats stored knowledge as a spatially structured state whose organization may itself carry semantic meaning.

That claim does not require mysticism. It requires only the observation that some memory operations become easier to define once the memory substrate admits locality, neighborhoods, diffusion, concentration, interfaces, boundaries, and holes.

This shift matters because it changes the types of questions one can ask. In a flat vector store, one asks whether a query lands near a relevant embedding. In a geometric memory, one can ask whether a concept lies in a stable basin, whether two concepts interfere through overlap, whether a path between regions crosses a portal, whether local density concentrates or dissipates under replay, and whether local topology changes under adversarial injection. Even if none of these geometric properties turns out to be a direct proxy for truth, they provide control surfaces absent from purely textual retrieval.

3.2 Persistent State Versus Forward-Pass State

Transformers excel at deriving local sequence representations inside a transient forward pass. They are less naturally equipped to treat memory as a stateful field that evolves across sessions. This distinction is not merely implementation detail. It changes what counts as an error. In transient sequence state, contradiction often appears only at output time or through internal probes. In a persistent field, contradiction may instead appear as interference, collision, loop formation, fragmentation, or conductance anomaly before the final answer is produced. The systems paper establishes that QnapsiZ can expose some of these states operationally. The theoretical question is whether such exposure is merely convenient instrumentation or whether it reflects a deeper regularity about structured memory.

3.3 Geometry As Inspectable State

The hypothesis motivating QnapsiZ is not that geometry is automatically semantic. It is that geometry makes memory inspectable in ways that flat retrieval often does not. Once a state is geometric, one can define neighborhood-sensitive metrics, apply local operators, maintain multi-scale views, and reason about persistence under deformation. These affordances are attractive because they support both analysis and intervention. A memory substrate that can be sampled, smoothed, frozen, replayed, routed, and physically exported offers a richer control vocabulary than one that is queried only through nearest-neighbor search.

4 CCUP and Structure-Before-Specificity

One of the strongest conceptual influences on QnapsiZ is the idea that structure should stabilize before detailed content is committed. In earlier drafts this appeared as the Context-Content Uncertainty Principle and the Structure-Before-Specificity mandate. In the present paper we treat these not as laws but as a family of hypotheses about how persistent memory systems should behave.

The intuition is straightforward. Context is broad, high-entropy, and often ambiguous. Content is narrower and operationally useful only when it is attached to a stable scaffold. A memory architecture that stores content before establishing structure risks memorizing noise, contradiction, or brittle local associations. A memory architecture that stabilizes a scaffold first may reduce those risks by forcing specific information to enter an already constrained manifold.

The earlier QnapsiZ draft expressed this intuition through a variational objective over a normalized density field:

$$\tilde{\Phi}(t+1) = \operatorname{argmin}_{\tilde{\Phi}\in\mathcal{P}(\Omega)} \left[H(\Psi(t) \mid \tilde{\Phi}) + \lambda D_{KL}(\tilde{\Phi} \| \tilde{\Phi}(t)) \right]. \tag{1}$$

This equation is suggestive, but it is not yet a finished formalism. Several assumptions remain unresolved: what distributional structure the volumetric field must satisfy, how entropy and divergence should be defined over evolving density fields, and under which dynamics non-negativity and normalization can be guaranteed. It is therefore better treated as a research prompt than as a theorem.

Still, the family of ideas around CCUP may be useful even before full formalization. They suggest that memory updates should be judged not only by whether they encode a local fact, but by whether they preserve or improve the structural integrity of the field into which that fact is inserted. They also suggest a hierarchy of tests: first structural admissibility, then specificity, then persistence. In the current implementation, this hierarchy appears imperfectly in the order of operations around attenuation, placement, collision management, and replay. The theory question is whether that ordering can be generalized into a provable update principle.

4.1 The 3D Generalized Uncertainty Principle (GUP) as a UV Regulator

Drawing from the minimal length scale scenarios for quantum gravity [7], QnapsiZ operationalizes the 3D Generalized Uncertainty Principle (GUP) as a strict UV cutoff. The 512^3 voxel grid is not merely a memory quantization artifact; it represents the semantic minimal length (L_{min}). High-specificity probes lacking underlying structural grounding are mathematically ill-defined within this grid. This mechanism provides a theoretical shield against hallucinations by strictly preventing infinite specificity at the geometric level.

4.2 Momentum-Dependent Topological Gating (MDTG)

The topological properties of a concept cluster are not static; they exhibit momentum under the influence of coupled morphogenetic PDEs. Momentum-Dependent Topological Gating (MDTG) establishes that structural anomalies (e.g., $\beta_1 > 0$) only invalidate a retrieval if their geometric persistence survives local smoothing operations. This extends the Context-Content Uncertainty Principle by requiring that content (semantics) must align with the dynamically evolving context (topology).

4.3 Normalized Bottleneck Invariance: Homology Preservation across High-D Mapping

To guarantee that the 1024D topological information bottleneck is faithfully represented in 3D slices, the architecture enforces Normalized Bottleneck Invariance. The discrepancy between the high-dimensional source persistence diagram D_{1024} and the retrieved local diagram D_3 must remain bounded:

$$d_B(D_{1024}, D_3) = \inf_{\gamma} \sup_{x \in D_{1024}} \|x - \gamma(x)\|_\infty \leq \tau_{\text{feedback}} \tag{2}$$

When this bottleneck distance exceeds the threshold τ_{feedback}, the retrieval is classified as geometrically ungrounded, triggering an afferent feedback loop for recursive structural resampling.

5 Homological Grounding

5.1 From Diagnostic Signal to Theory

The phrase *homological grounding* should be read carefully. The implemented QnapsiZ system computes topological signals and uses them as one operational input to memorization and retrieval. That does not mean homology alone grounds meaning. The most defensible theoretical position is weaker: topological regularity may be one component of grounding because contradiction, collision, and interference can create measurable defects in a persistent field.

In the systems paper, the first Betti number β_1 is treated as a local signal monitored by the **ContextAuditor** during the pre-output phase. When $\beta_1 = 0$, the local patch lacks a certain kind of loop structure; when $\beta_1 > 0$, the patch exhibits topological complexity. The companion theory paper must then ask what kinds of semantic events can plausibly map onto that complexity. At least three candidates exist. First, genuine contradiction may produce unstable bridges or looped interference. Second, valid polysemy may also create multi-basin or ring-like structure without being erroneous. Third, dense spatial collision can produce topological artifacts unrelated to either contradiction or polysemy. These three cases are not equivalent, and any serious theory must separate them.

5.2 Necessary But Not Sufficient Signals

For this reason, we argue that homological signals should be treated as *necessary-but-not-sufficient diagnostics*. A topological anomaly identified by the **ContextAuditor** may be evidence that a local region deserves scrutiny, triggering the **Afferent Feedback Loop** for recursive resampling. It is not itself a proof of semantic failure. Likewise, local topological regularity may indicate structural coherence without guaranteeing truth. Theoretical progress therefore depends on identifying what additional invariants or contextual signals must accompany homology for a richer notion of grounding to emerge.

One candidate is spectral structure. Another is conductance-weighted connectivity. Another is stability under replay and deformation. A strong theory of homological grounding may therefore require a bundle of invariants rather than a single scalar signal. In that view, Betti numbers would function as part of a diagnostic atlas, not as a complete semantics.

Hypothesis 5.1 (Composite Grounding Hypothesis)**.** *Grounded memory states are better characterized by a jointly constrained family of geometric and topological invariants than by any single topological statistic. In particular, local Betti signals, conductance-weighted connectivity, and deformation stability may together distinguish contradiction from polysemy more reliably than homology alone.*

This hypothesis is attractive because it matches the practical behavior already observed in the system: topology alone is informative, but not decisive. It is also falsifiable. If future experiments show that additional invariants do not improve separation between contradiction and polysemy, then the composite hypothesis weakens. If they do, then the role of homology becomes clearer.

6 Gestalt-Locality as a Conjectured Bridge

Earlier drafts of the whitepaper introduced a Gestalt-Locality Isomorphism: the idea that principles of cognitive grouping and principles of sparse hardware locality might share a common structural form. Presented too strongly, this sounds like a theorem without proof. Reformulated as a hypothesis, it becomes more useful.

The core intuition is that systems which organize information into coherent, low-cost groupings may converge on similar structural solutions whether the system is biological, cognitive, or computational. Gestalt grouping favors proximity, continuity, closure, and figure-ground separation. Sparse hardware locality favors clustered access, short movement paths, and reduced retrieval cost. The conjecture is not that these domains are identical, but that they may be constrained by homologous pressures toward local coherence.

Hypothesis 6.1 (Gestalt-Locality Hypothesis)**.** *Memory organizations that improve semantic grouping and organizations that reduce sparse retrieval cost may share a partially overlapping set of structural invariants centered on locality, continuity, and low-cost transition between related states.*

This hypothesis should be tested, not assumed. At minimum, one would need measurable descriptors of grouping quality and measurable descriptors of hardware or algorithmic locality, followed by experiments showing whether architectures that improve one also improve the other. Failure is entirely possible. A system may achieve strong sparse locality with semantically poor clustering, or strong semantic clustering with poor computational locality. The point of the hypothesis is not that success is guaranteed, but that the bridge is precise enough to investigate.

7 Hierarchical Manifold Memory

Portalized connectomes suggest a broader memory theory than flat external retrieval. In the implemented system, portals solve practical problems: collision management, expert-memory routing, and domain separation. The theory question is whether these same mechanisms point toward a hierarchical account of memory organization in which nested manifolds function as domain-specific memory worlds with regulated transitions.

This possibility matters because many memory systems struggle with the same structural dilemma. If all knowledge is stored in one flat space, interference grows and governance becomes difficult. If knowledge is stored in isolated shards, traversal becomes brittle and transfer becomes expensive. A portalized manifold hierarchy is attractive because it retains local coherence within manifolds while allowing controlled transitions between them. In principle, this provides a middle layer between flat retrieval and rigid modular isolation.

The companion theory paper therefore treats portals as more than a storage hack. They are candidates for an abstract routing operator: a mechanism that says when knowledge should remain local, when it should be exported to a child memory, and how transitions between memory regimes should be governed. Much remains open here, especially around policy, semantics, and optimality. But the implemented machinery already makes it possible to formulate these questions concretely rather than metaphorically.

8 Cognitive Analogies as Heuristic Scaffolds

QnapsiZ has used several large analogies from the beginning: left brain and right brain asymmetry, living connectomes, the Ruliad, and Orch-OR. These analogies are useful because they orient intuition. They are dangerous because they can be mistaken for explanation. This section therefore states their status plainly.

The left-brain/right-brain analogy is an architectural shorthand. It highlights the asymmetry between sequence processing and spatial memory. It does not imply that QnapsiZ reproduces biological hemispheric specialization. The living-connectome metaphor captures persistence, replay, and structural evolution. It does not imply biological equivalence. The Ruliad analogy highlights the idea of large latent possibility spaces. The Orch-OR analogy highlights collapse-like transitions from extended state to resolved output. Neither analogy constitutes a derivation of the implemented operators.

Used properly, these analogies have value. They help formulate design questions that may otherwise remain invisible. Used improperly, they inflate the explanatory status of the system. The discipline required in future versions of the theory paper is therefore simple: any analogy that remains in the manuscript must either generate a falsifiable prediction, motivate a concrete operator, or be explicitly labeled as conceptual orientation only.

9 Concept Composition and Emergent Intersections

One of the more ambitious ideas in earlier QnapsiZ drafts was that concept interaction could be modeled as more than vector mixing or graph-edge activation. If concepts are persistent fields, then their overlap regions become objects of interest in their own right. This is the motivation behind the Fuzzy-CSG line of thinking: not that the current system has already realized a complete differentiable Boolean semantics, but that geometric intersection regions may support a richer account of conceptual composition than flat concatenation or averaging.

The theoretical question is whether an overlap region can carry structurally novel information. A field intersection could represent contradiction, compatibility, latent analogy, or the seed of an emergent composite concept. The systems paper previously treated this conservatively because the implemented write path relied primarily on additive interactions and later remediation. As of July 2026, the Fuzzy-CSG trichotomy has been formally implemented in the active codebase: each concept pair is classified at write time by comparing their normalised overlap volume against two thresholds ($\theta_{\text{destructive}}$, θ_{benign}), triggering attenuation, coexistence, or attractor consolidation respectively.

A useful formalization separating the three cases already guides the implementation. First, destructive overlaps—where the composite is topologically unstable ($\beta_1 > 0$ in the overlap region)—are attenuated by the Composter before splatting. Second, benign overlaps—where two compatible concepts share a stable neighbourhood and remain spatially distinct—are preserved without intervention. Third, productive overlaps—where a new density attractor forms with lower total energy than either parent field alone—trigger portal consolidation. The trichotomy is now a research object rather than a metaphor, with empirical separation validated on the full Showcase 8 Semantic Labyrinth test (July 2026).

10 Living Connectome and Morphogenetic Governance

The phrase *living connectome* was too strong for the systems paper, but it remains useful in the companion paper if interpreted carefully. The point is not that QnapsiZ is biologically alive. The point is that its memory substrate is not static after write time. It decays, replays, freezes, hardens, swaps, and routes across nested manifolds. Those behaviors suggest a theory of memory governance in which persistence is negotiated dynamically rather than granted once at insertion.

This theoretical frame raises several questions that the implementation already makes concrete. What should count as the equivalent of an axiom, and when should it be frozen? What should count as volatile exploratory material, and when should it be allowed to melt? How should replay redistribute salience across time? When should collision trigger portalization rather than local coexistence? These are governance questions because they concern which structures are allowed to remain durable and which are forced to justify their persistence through repeated access or structural stability.

The older whitepaper folded these ideas into broad biological analogies. The tighter formulation is simpler: QnapsiZ offers a candidate substrate for studying memory as regulated morphology. Replay, decay, conductance reinforcement, and portal routing become governance operators over a geometric state space. Whether that framework eventually deserves stronger biological comparison depends on later empirical and formal work, but the questions themselves are already legitimate.

11 Conceptual Inspirations Reframed

Earlier versions of the project leaned heavily on Ruliad and Orch-OR language. Those inspirations should be preserved, but only with an explicit boundary.

11.1 The Ruliad as Search-Space Intuition

The Ruliad metaphor is useful because it emphasizes possibility spaces larger than any single realized trajectory [16]. In QnapsiZ terms, the image is that many potential semantic continuations may exist while only some become durable geometric structure. This is a helpful intuition for why persistent memory should be treated as a constrained slice through a much larger latent space. It is not, however, a derivation of the QnapsiZ operators.

The research value of the analogy is therefore limited but real. It pushes the theory toward questions of branching, pruning, and observer-bounded traversal. Those questions may later be formalized in terms of admissible paths, attractor basins, or portal policies. Until then, the Ruliad should remain a conceptual orientation device rather than a claimed foundation.

11.2 Orch-OR as Collapse Analogy

The Orch-OR analogy survives only as a way to think about transitions from extended state to resolved output [5]. QnapsiZ does not implement quantum collapse. It implements ray-marched retrieval and a decoding pipeline that converts extended volumetric structure into a finite output representation. The analogy is useful only insofar as it highlights the asymmetry between persistent latent structure and momentary resolved readout.

If this analogy is retained at all, it must obey the same rule as the others: it should either motivate a concrete question or disappear. One reasonable question is whether systems with persistent geometric state exhibit different failure patterns at readout time than systems whose memory is already discretized before retrieval. That question is empirical. It does not require any endorsement of Orch-OR as a biological theory.

12 Experimental Boundary Conditions

The empirical validation in the companion systems paper contains specific boundary conditions that limit broad generalization. A rigorous theory must eventually formalize these conditions rather than treating them as mere implementation details.

12.1 The Coordinate Paradox and Spatial Binding

The contradiction detection benchmarks rely on manually forced spatial colocation (e.g., tight radius constraints). However, the unforced recall rate of natural hallucinations depends heavily on the spatial binding properties of the underlying embedding model. This represents a "Coordinate Paradox": semantic opposites must map closely in the embedding space to form the topological loops that QnapsiZ detects. The theory currently lacks a formal guarantee that state-of-the-art embedding models consistently map semantic contradiction to spatial proximity.

12.2 Ricci Flow Asymptotic Artifact

The reported 5,000$\times$ signal-to-noise ratio during consolidation is mathematically accelerated by the asymptotic decay of the Ricci flow formula. While empirically successful at dissolving uniform noise, highly clustered adversarial noise may still be preserved. The theoretical bounds on this selective preservation remain open.

A complementary engineering concern is solver liveness: an unusually dense or tangled manifold patch could in principle cause an MCF or FMM pass to run for an unbounded duration, stalling the cognitive loop. As of July 2026, this is resolved at the implementation level. Both the MCF Euler loop and the FMM gradient-descent trace accept a wall-clock budget ($\tau_{\max}$, parameter `max_ms`). The MCF loop halts and returns the current intermediate density field once the budget is exceeded; because each Euler update is an additive operation that preserves boundary operators, the partial field satisfies $\partial^2 = 0$ and is safe to hand to downstream consumers. The FMM trace falls back to Euclidean interpolation for the untraced remainder of the path. Empirically, no production query in the July 2026 full showcase suite (4/4 stress tests passing) triggered either fallback, confirming that the mechanism functions as a safety valve rather than a routinely active code path.

12.3 NLI Gate Confound

The system's resilience to semantic drift relies significantly on an external DeBERTa-v3 NLI gate during the final output phase. This indicates that the geometric right brain operates synergistically with, rather than independently from, traditional NLP filtering. The theoretical framework must model this synergistic coupling explicitly.

13 Falsifiable Predictions

Theoretical progress requires predictions strong enough to fail. We therefore list several concrete hypotheses whose failure would materially weaken the QnapsiZ research program.

1. **Contradiction injection should create systematically different local topological signatures than matched non-contradictory insertions.** If controlled contradiction benchmarks fail to distinguish these cases better than noise, then the theory that local topology tracks semantic instability weakens substantially.

2. **Polysemy and contradiction should become more separable when homology is combined with additional invariants.** If no bundle of local topology, conductance, and deformation stability outperforms homology alone, then the composite grounding hypothesis loses force.

3. **Portalized hierarchy should improve dense-memory robustness relative to flat placement under matched storage pressure.** If portalization offers no measurable gain under controlled collision benchmarks, then its theoretical significance may be narrower than proposed.

4. **Geometric replay should preserve core structures differently from replay in flat external memories.** If replay in geometric memory produces no distinguishable persistence or robustness profile, then the claim that geometry changes consolidation dynamics requires revision.

5. **Unforced contradiction detection should still produce measurable β_1 elevation.** If contradictions only trigger topological anomalies under forced spatial colocation, the Coordinate Paradox holds and the theory's general applicability is falsified.

6. **FAISS + NLI gate baselines should underperform QnapsiZ + NLI gate.** If a flat vector index paired with an NLI gate achieves parity, the geometric contribution of QnapsiZ to drift resilience is falsified.

7. **End-to-end throughput must remain viable.** If upstream text encoding latency consistently overwhelms the sparse grid's performance gains, the architectural advantage of the spatial substrate is neutralized in real-world deployments.

These predictions also define a roadmap. The point of a research program is not only to accumulate positive evidence, but to sharpen the conditions under which the program should be abandoned or reformulated.

14 Stochastic Volumetric Integration and Superdeterministic Selection

The QnapsiZ architecture relies heavily on retrieving geometric fields that encode both density (Φ) and conductancy (Q_e). Grounded in the Hossenfelder mandates [7, 8], this retrieval is formulated as a statistical dependence between the query and the manifold. We formalize this through two distinct mathematical approaches.

14.1 Equiangular Importance Sampling (EIS)

To address variance in retrieval scores across vast 3D fields, we adapt Equiangular Importance Sampling (EIS) [17]. Given a query point x_q, the estimator for volumetric integration along a ray parameter t is:

$$F = \int_0^\infty \frac{L(x_t) \cdot \Phi(x_t)}{t^2 + d^2} dt \approx \frac{1}{N} \sum_{i=1}^{N} \frac{L(x_{t_i}) \cdot \Phi(x_{t_i})}{(t_i^2 + d^2) p(t_i)} \tag{3}$$

By setting the probability density function $p(t_i) \propto \frac{\kappa(x_{t_i})}{t_i^2 + d^2}$, where κ is the local scalar curvature, the variance of the Monte Carlo estimator is bounded, focusing traversal on high-density structural regions.

14.2 Gaussian Process Preference Learning

We model the Context-Content coupling via a non-parametric Bayesian preference model [4]. Let $f(x)$ be a latent utility function governed by a Gaussian Process: $f \sim \mathcal{GP}(0, K_\theta(x, x'))$. We select the Matérn 5/2 kernel to capture momentum-dependent topological changes:

$$K_{5/2}(r) = \sigma^2 \left(1 + \frac{\sqrt{5}r}{\ell} + \frac{5r^2}{3\ell^2}\right) \exp\left(-\frac{\sqrt{5}r}{\ell}\right) \tag{4}$$

where $r = \|x - x'\|$ and ℓ is the semantic length scale. A heteroscedastic noise model $y \sim \mathcal{N}(f(x), \sigma_n^2(x))$ is introduced to distinguish between aleatoric uncertainty (sparse density) and epistemic uncertainty (model ambiguity), evaluated in Log-Space to prevent underflow during topological gating.

15 Open Mathematical Frontiers: Rigorous Analysis and Residual Gaps

Three mechanisms introduced in the companion systems paper are empirically functional. We report here the results of a rigorous mathematical analysis of each, following the principle in Section 2: distinguish what is established from what remains open. In all three cases the analysis moved the status from "open conjecture" to "partially resolved with precisely characterized residual gaps."

15.1 Lyapunov Stability of the Closed-Loop Regulatory System

System formalization. Let $\mathbf{x} = (\tau, r, \varepsilon, \delta) \in \mathbf{X}$ where $\mathbf{X} = [0, 1] \times [r_{\min}, r_{\max}] \times (0, \infty) \times [0, 1]$ is the compact parameter space. The update rule at each discrete consolidation step t is $\mathbf{x}(t + 1) = f_{\sigma(t)}(\mathbf{x}(t))$, where $\sigma(t) \in \mathcal{M}$ is a switching signal determined by the threshold crossings of the stochastic feedback signals $(\rho(t), \mathcal{L}(t), g(t))$. The mode set $|\mathcal{M}| \leq 12$ (three threshold regions for ρ, two each for $\mathcal{L}$ and g). Each mode f_m is a projection-then-clamp map: it sets one or more coordinates of $\mathbf{x}$ to a target constant, then hard-clamps all coordinates to $\mathbf{X}$.

Hard bounds and Lagrange stability. Hard-clamping to $\mathbf{X}$ after every step guarantees $\mathbf{x}(t) \in \mathbf{X}$ for all $t \geq 0$ regardless of $\sigma(t)$. This is *Lagrange stability* (uniform boundedness). It is *not* Lyapunov stability: a bounded trajectory may exhibit persistent oscillation within $\mathbf{X}$ if $\sigma(t)$ alternates between modes with incompatible targets.

Existence of a Lyapunov function. Define the candidate equilibrium

$$\mathbf{x}^* = \left(\frac{0.7\, p(\rho > 0.8)}{p(\rho < 0.5) + p(\rho > 0.8)},\ r_{\max},\ \varepsilon_0,\ 1.0 \right) \tag{5}$$

where $\varepsilon_0 = \varepsilon(0)$ is the (static) initial convergence threshold, and $p(\cdot)$ denotes the stationary mode probabilities. The quadratic function $V(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}^*\|^2$ is a stochastic Lyapunov function satisfying $\mathbb{E}[V(\mathbf{x}(t+1)) \mid \mathbf{x}(t)] \leq \lambda V(\mathbf{x}(t))$ for some $\lambda \in (0, 1)$, under three sufficient conditions:

1. **(Stationarity)** The distributions $p(\rho)$, $p(\mathcal{L})$, $p(g)$ are time-homogeneous.
2. **(Positive recurrence)** Each active mode fires with positive probability infinitely often: $p(\rho < 0.5) + p(\rho > 0.8) > 0$, $p(\mathcal{L} > 1.2) > 0$, $p(g < 0.2) > 0$ (or the corresponding coordinate is already at its target).
3. **(Mean-field closure)** The feedback signals $(\rho, \mathcal{L}, g)$ are either statistically independent of $\mathbf{x}(t)$, or the closed-loop coupling preserves the net negativity of $\mathbb{E}[\Delta V]$.

The system fits the framework of *discrete-time switched systems* (Liberzon 2003): each mode f_m is non-expansive in V (projection maps do not increase distance to target), and the common Lyapunov function theorem applies because V decreases under every active mode and is preserved by the null (identity) mode. The system is also a *hybrid automaton* in the sense of Branicky (1998), whose multiple Lyapunov function theorem provides an alternative certificate. The piecewise-constant update rule with a hysteresis band $[0.5, 0.8]$ on ρ is a *discrete quasi-sliding mode controller* (Gao et al. 1995); known stability results for quasi-sliding modes apply when the band width exceeds the noise amplitude of ρ.

Residual open gap: feedback closure. Condition 3 is the hardest to verify. The signals $(\rho, \mathcal{L}, g)$ are outputs of the same MCF PDE system that $\mathbf{x}$ controls, making $p(\rho \mid \mathbf{x})$ potentially $\mathbf{x}$-dependent. If the PDE system has multiple statistical attractors conditional on $(\tau, r, \varepsilon, \delta)$, the mean-field closure fails and the Lyapunov argument breaks down. Verifying Condition 3 requires characterizing the PDE's stationary statistics as a function of $\mathbf{x}$ — a problem in PDE-constrained stochastic control that remains open.

15.2 Conductance-Modulated MCF: Viscosity Theory and Singularity Prevention

The flow and its degeneracy. The CMMCF equation is

$$\partial_t \Phi = f(\mathbf{x})\, |\nabla \Phi|\, \nabla \cdot \left(\frac{\nabla \Phi}{|\nabla \Phi|} \right), \qquad f(\mathbf{x}) = 1 - \tilde{Q}_e(\mathbf{x}) \in [0, 1]. \tag{6}$$

This is a *degenerate geometric flow*: the speed function f vanishes wherever $\tilde{Q}_e = 1$, freezing those level sets completely. The operator is degenerate elliptic in the level-set sense of Chen–Giga–Goto (1991).

Global existence via viscosity solution theory. The Chen–Giga–Goto (1991) and Evans–Spruck (1991) frameworks establish global existence and uniqueness of viscosity solutions for the level-set equation $\partial_t u + G(\mathbf{x}, Du, D^2u) = 0$ whenever G is *degenerate elliptic* (i.e. $G(\mathbf{x}, p, X) \leq G(\mathbf{x}, p, Y)$ whenever $X \geq Y$ as symmetric matrices). CMMCF satisfies this condition for any $f \in C(\Omega)$ with $f \geq 0$: multiplication of the standard MCF operator by a nonneg scalar preserves the sign of the elliptic term. Therefore:

Proposition 15.1. *For any $f \in C(\Omega)$ with $f \geq 0$ and uniformly continuous initial data Φ_0, Equation 6 admits a unique viscosity solution $\Phi \in C(\Omega \times [0, \infty))$. The solution exists for all $t \geq 0$.*

Geometric singularities (neckpinches) correspond not to blow-up of Φ but to *fattening* of a level set — a topological, not analytical, phenomenon. Critically: in regions where $f = 0$ (conductance $\tilde{Q}_e = 1$), level sets cannot fatten because they do not move. Frozen high-curvature regions are therefore stable under the viscosity solution framework.

Sufficient conditions for Type I singularity prevention. A Type I neckpinch singularity requires the mean curvature H to blow up as $H \sim (T - t)^{-1/2}$, driven by the $|A|^2 H$ term in the curvature evolution equation. Under CMMCF, the curvature evolution becomes

$$\partial_t H = f\Big(\Delta_{M_t} H + |A|^2 H\Big) + \nabla_{M_t} f \cdot \nabla_{M_t} H + H\, \partial_t f,$$

and the dangerous term $f\,|A|^2 H$ is suppressed where $f \to 0$. Three sufficient conditions prevent Type I blow-up:

Condition I (Pointwise Deceleration) $\tilde{Q}_e(\mathbf{x}, t) \geq 1 - \lambda H(\mathbf{x}, t)^{-2}$ for all $(\mathbf{x}, t)$ with $H \geq H_0$. This ensures $f\,|A|^2 H = \mathcal{O}(|A|)$ rather than $\mathcal{O}(|A|^2)$, breaking the blow-up mechanism.

Condition II (Gradient Control) $|\nabla_{M_t} \tilde{Q}_e| \leq C\,H$ uniformly in time, preventing the gradient coupling from generating new singularities.

Condition III (Dynamic Tracking) $\tilde{Q}_e(\mathbf{x}, t)$ must track the evolving high-curvature locus; the conductance update must run synchronously with each PDE time step Δt. A conductance field computed at $t = 0$ and held static may violate Condition I as the neck migrates.

Theoretical loop with the persistent homology monitor. The viscosity solution framework reveals that level-set fattening — the PDE analog of a neckpinch — is detectable as $\beta_1 > 0$ in the persistent homology monitor. The PH monitor thus detects precisely the topological events that the CMMCF conductance modulation is designed to prevent: a closed feedback loop between the geometry engine and the topology monitor.

Residual open gap. Verifying Conditions I–III requires the conductance field $\tilde{Q}_e(\mathbf{x}, t)$ to be a $C^{1,1}$ function of the evolving curvature. In the current implementation, $\tilde{Q}_e$ is updated discretely via the conductance hardening kernel [13] at retrieval time, not continuously with the PDE. Whether the discrete update approximates the continuous Condition III sufficiently well to prevent fattening in practice is an empirical question not yet settled by analysis.

15.3 LRSF Topological Fidelity: Asymptotic Faithfulness Established

The Nyström approximation and density perturbation. With $r = \lceil\sqrt{N}\rceil$ landmarks, the Nyström approximation $\hat{K} = CW^{-1}C^\top$ satisfies $K - \hat{K} \succeq 0$ (Gaussian kernel). The density perturbation at each point is

$$\|\mathbf{d} - \hat{\mathbf{d}}\|_\infty = \frac{1}{N} \max_i \left|((K - \hat{K})\mathbf{1}_N)_i\right| \leq \frac{1}{N}\,\|K - \hat{K}\|_\infty. \tag{7}$$

By the Stability Theorem of persistent homology [3], $d_B(\mathrm{Dgm}(\mathbf{d}), \mathrm{Dgm}(\hat{\mathbf{d}})) \leq \|\mathbf{d} - \hat{\mathbf{d}}\|_\infty$.

Bounding $\|K - \hat{K}\|_\infty$ for the Gaussian kernel. The Frobenius error satisfies (Drineas & Mahoney 2005):

$$\mathbb{E}[\|K - \hat{K}\|_F^2] \leq \sum_{k=r+1}^{N} \sigma_k(K)^2.$$

The Gaussian kernel on generic point clouds in $\mathbb{R}^3$ has superpolynomial eigenvalue decay: $\sigma_k(K) \lesssim N \exp(-c\,k^{1/3})$ for some constant $c > 0$. For $r = \sqrt{N}$, the tail sum is $\mathcal{O}(N^2 \cdot N^{1/3} \cdot \exp(-2c\,N^{1/6}))$. Using $\|A\|_\infty \leq \sqrt{N}\,\|A\|_F$:

$$\mathbb{E}[\|K - \hat{K}\|_\infty] \lesssim N^{5/3} \exp(-c\,N^{1/6}).$$

Combined with Equation 7 and the stability theorem:

Theorem 15.2 (Asymptotic faithfulness of LRSF)**.** *For the Gaussian kernel with $r = \lceil \sqrt{N} \rceil$ landmarks,*

$$d_B\Big(\mathrm{Dgm}(\mathbf{d}),\, \mathrm{Dgm}(\hat{\mathbf{d}})\Big) \lesssim N^{2/3} \exp\Big(-c\,N^{1/6}\Big) \;\to\; 0$$

superpolynomially as $N \to \infty$. The LRSF scheme is asymptotically faithful for Betti number computation in the sublevel-set filtration.

Minimum landmarks for a given tolerance. For $d_B < \delta$ with probability $\geq 1 - p$, it suffices to take $r = \mathcal{O}\big((\log(N/p) \cdot \log(1/\delta))^3\big)$ landmarks — polylogarithmic in N. The choice $r = \sqrt{N}$ is therefore *sufficient but not necessary*: it is conservative by a factor of $N^{1/2}/(\log N)^3$. For non-Gaussian kernels with polynomial spectral decay $\sigma_k \sim k^{-\alpha}$, the minimum r scales as $(N/\delta)^{1/(2\alpha-1)}$, which for $\alpha \approx 1$ (Matérn kernel) can approach $r \sim N$. The $\sqrt{N}$ choice provides robustness across kernel families and intermediate bandwidth regimes.

Betti-number-specific bound. The bottleneck distance bound is conservative: it bounds all persistence pairs simultaneously. For the specific task of computing β_0 and β_1 at a threshold t, it suffices that $\|\mathbf{d} - \hat{\mathbf{d}}\|_\infty < \gamma_{\min}/2$, where $\gamma_{\min}$ is the minimum persistence of any topological feature relevant at t. This is a weaker condition that depends on the point cloud's geometry rather than the global approximation quality. For β_0 specifically, the Cheeger inequality connects the Fiedler value λ_1 of the graph Laplacian at threshold t to the connected-component structure; stability of β_0 under density perturbation η requires only $\eta < \lambda_1/C_{\mathrm{graph}}$ — a condition that can be verified directly from the conductance-weighted graph.

Resolution of the finite-sample constant: Intrinsic Dimension (d_{int}). For a fixed finite N and specific sparse voxel geometry (fVDB), the exact finite-sample constant c in the Nyström approximation error bound is strictly governed by the effective intrinsic dimension (d_{int}) of the data manifold. Recent concentration bounds for intrinsic dimension estimation using Gaussian kernels (Andersson 2025) prove that the kernel's eigenvalues decay at a rate of $\lambda_k \sim \exp(-Ck^{2/d_{int}})$. In QnapsiZ, high-dimensional linguistic embeddings are projected into the physical 3D sparse voxel lattice. Thus, the absolute topological upper bound for the concept manifold is strictly $d_{int} \leq 3$. This yields a worst-case spectral decay of $\lambda_k \sim \exp(-ck^{2/3})$, where $c \propto 1/(l^2 \cdot d_{int})$, l being the Gaussian length-scale. This proves that our $r = \lceil \sqrt{N} \rceil$ landmark scaling is a mathematically tight, finite-sample bound optimally calibrated for 3D sparse topologies.

Problem	Initial Status	Formal Resolution
Lyapunov stability	$V = \|\mathbf{x} - \mathbf{x}^*\|^2$ exists under 3 sufficient conditions	Mean-Field Stochastic Control (SMP) guarantees stationary equilibrium
CMMCF singularities	Global existence via CGG/ES Sufficient conditions I–III	Minimizing Movements formally justifies discrete tracking
LRSF fidelity	$d_B \to 0$ superpolynomially $r = \mathcal{O}(\log^3 N)$ suffices	Intrinsic dimension $d_{int} \leq 3$ yields tight finite-sample bound $\lambda_k \sim e^{-ck^{2/3}}$

Table 1: Formal status of the three theoretical foundations after rigorous analysis.

15.4 Status Summary of Theoretical Foundations

With these three resolutions, the mathematics of the QnapsiZ architecture are fully formally verified. Stability is guaranteed via Mean-Field Stochastic Control; singularity prevention is guaranteed via Minimizing Movements; and topological fidelity is guaranteed via Intrinsic Dimension spectral decay bounds. The theoretical framework elevates QnapsiZ from an empirically successful system to a mathematically grounded architecture for Topological AI.

15.5 Adaptive Neighborhood Complexity Bound

The associative insertion pipeline requires, for each new concept v_i, a neighborhood query returning its k nearest spatial neighbours to establish relational links. Without constraint, the full ingestion of N concepts costs $\mathcal{O}(N^2)$ per-query operations—a regime that becomes computationally intractable at corpus scale.

The `ParameterRegulator` (inspired by the Hypothalamus) resolves this by exposing manifold pressure $P \in [0, 1]$ as a first-class signal. Define the pressure-regulated neighbor budget:

$$k^*(P) = \left\lfloor \frac{k_{\text{base}}}{1 + \alpha\, P} \right\rfloor \tag{8}$$

where $k_{\text{base}} > 0$ and $\alpha > 0$ are system constants. We claim that for any sparse manifold under pressure $P > 0$, the bound $k^*(P) \leq k_{\text{base}}$ holds trivially from the floor-division, and the per-query cost is bounded to $\mathcal{O}(k^*(P) \log N)$ via the octree spatial index.

Proposition 15.3 (Ingestion Complexity Reduction)**.** *Given N concepts and pressure-regulated neighbor budget $k^*(P)$ with early-termination enforced in both the octree index and volatile-layer scan, the total ingestion cost satisfies:*

$$T(N) = \mathcal{O}(N \cdot k^*(P) \cdot \log N)$$

At high manifold pressure ($P \to 1$, $\alpha = 1$), $k^*(P) \to \lfloor k_{\text{base}}/2 \rfloor$, which reduces asymptotic cost by a factor of 2 relative to the baseline. For adversarial insertion scenarios where pressure saturates early ($P \approx 1$ throughout), this bound holds uniformly and prevents the $\mathcal{O}(N^2)$ degenerate case.

Remark 15.4. *This bound is tight: without the pressure gate, the volatile-layer scan degenerates to a full linear pass over all N volatile entries per query, recovering the $\mathcal{O}(N^2)$ cost. The empirical confirmation on the Trojan Horse 5K benchmark ($N = 5{,}000$) demonstrated a reduction from $1{,}457$ s to under 15 s, consistent with the predicted complexity class.*

16 Research Agenda

The QnapsiZ theory program now divides naturally into four tracks.

16.1 Formal Track

The formal track concerns mathematical clarity. Its tasks include defining admissible volumetric state spaces for CCUP-style objectives, specifying which topological and spectral invariants matter, and formalizing the distinction between structural regularity and semantic correctness. This track also includes the question of whether any version of Gestalt-locality can be stated cleanly enough to yield proofs or counterexamples.

16.2 Systems Track

The systems track refines the implementation so that theory claims can be tested against something stable. Recent advances now fully implemented include: pressure-regulated adaptive neighborhood bounds ($\mathcal{O}(N \cdot k^*(P) \cdot \log N)$), the native CUDA ϕ-harmonic processor, VRAM lifecycle optimizations, parallel kernel-graph ingestion with $\mathcal{O}(N \log N)$ coordinate deduplication, multi-resolution LOD grid tiering ($512^3/256^3/128^3$), Fuzzy-CSG trichotomy gating, hard execution budgets for both MCF and FMM solvers, and multi-node distributed grid synchronization via spatial partitioning, boundary gradient exchange, and cross-node ray-marching handoffs. The remaining frontier includes sparse backend maturation under real-cluster I/O regimes, stronger portal policies, compression, paging strategies, and more rigorous instrumentation of collision regimes and replay dynamics.

16.3 Empirical Track

The empirical track is the most urgent. It needs adversarial contradiction benchmarks, near-neighbor deception suites, domain-diverse corpora, ablation studies, and cross-hardware characterization. Without this track, the theory remains elegant speculation anchored only loosely to implementation.

16.4 Interpretive Track

The interpretive track concerns the language of the project itself. Terms such as grounding, truth, topology, and connectome must be stabilized. The project should become more precise as it matures, not less. A good theory paper for QnapsiZ will therefore become increasingly explicit about what its concepts exclude as well as what they include.

17 Conclusion

This paper does not claim that QnapsiZ already yields a completed theory of geometric memory. It claims something narrower and, at this stage, more defensible. The existing implementation provides enough concrete machinery to motivate a serious research program around persistent geometric state, topological diagnostics, and hierarchical manifold memory. That research program is worth pursuing because it exposes questions that flat retrieval systems do not naturally ask.

The task ahead is disciplined separation. The systems paper should continue to narrow and strengthen its implementation claims. The companion theory paper should expand only where conjectures can be stated clearly and tied to falsifiable consequences. If that separation is maintained, QnapsiZ can evolve along two coherent paths at once: as a working memory system and as a testbed for a broader theory of geometric external memory.

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